Coupled spin-charge drift-diffusion approach for a two-dimensional electron gas with Rashba spin-orbit coupling

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Abstract

Based on kinetic equations for the density matrix, drift-diffusion equations are derived for a two-dimensional electron gas with Rashba spin-orbit coupling. Universal results are obtained for the weak coupling case. Most interesting is the observation that with increasing spin-orbit coupling strengths there is a sharp transition between spin diffusion and ballistic spin transport. For strong spin-orbit coupling, when the elastic scattering time is much larger than the spin relaxation time, undamped spin-coherent waves are identified. The existence of these long-lived spin-coherent states is confirmed by exact analytical results obtained from microscopic kinetic equations valid in the ballistic regime.

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I. INTRODUCTION

One of the major challenges of spintronics is the generation and manipulation of a spin polarization by exclusively electronic means in nonmagnetic semiconductors at room temperature. A lively discussed phenomenon in this field is the spin-Hall effect [1, 2], the theoretical description of which is still very controversial. In the experimental analysis of this effect [2, 3, 4, 5], an electric-field induced spin accumulation is observed near the edges of the sample. A widespread idea to understand this phenomenon relies on the notion of a spin current oriented transverse to the applied electric field. [1, 2] This current is assumed to induce a spin accumulation in the spin-Hall experiment. However, this seemingly clear physical picture became the subject of serious controversies in the literature. [6, 7, 8, 9, 10, 11, 12, 13, 14] The main problem results from the fact that the spin is not conserved in spin-orbit coupled systems so that the definition of a related spin current becomes ambiguous. Alternative approaches seem to be more suitable, which avoid the identification of an appropriate spin current. Most profound approaches of this kind start from a microscopic theory for the spin-density matrix [14, 15, 16, 17, 18, 19, 20, 21, 22] or Keldysh Green functions [23, 24], followed by an analysis of its long-wavelength and low-frequency limit. These treatments provide a complete physical description of spin-related phenomena including accumulation, diffusion, and relaxation of spin as well as magnetoelectric and spin-galvanic effects. [19] Based on this approach, it was recently concluded that as soon as spins are polarized, the spin-orbit interaction leads to spin precession and related spatial oscillations of the spin accumulation. [25] A specific difficulty of the phenomenological drift-diffusion approach is the formulation of appropriate boundary conditions [26, 27] to solve the differential equations for the position and time dependent spin and charge densities.

Another challenge in the field of spintronics refers to mechanisms that allow long spin relaxation times. In many situations of practical relevance, the main spin relaxation mechanism is due to spin-orbit interaction (Dyakonov-Perel spin relaxation), which leads to an effective internal magnetic field that causes spin precession in the plane perpendicular to the magnetic field. Momentum scattering on impurities randomly reorient the respective precession axis so that an averaged spin dephasing results. From an application point of view, the suppression of this spin dephasing in spintronic devices is a demand of high priority. An enhancement of the spin relaxation time has been predicted to occur in spatial

regions near the edges of a spin-polarized stripe. [28] A suppression of spin relaxation is also observed in samples with an initial periodic spin profile. [28] The related spin-coherent standing wave has a several times longer spin relaxation time than a homogeneous electron spin polarization. The phase as well as the long spin relaxation time of these novel spin waves could be important to reading, writing, and manipulating information in spintronic devices.

In this paper, we focus on coupled spin-charge drift-diffusion equations, which allow a treatment of spin-related phenomena that are interesting both from a theoretical and experimental point of view. Starting from microscopic transport equations for the spin-density matrix of a two-dimensional electron gas (2DEG) with spin-orbit coupling of the Rashba type, we derive in an exact manner drift-diffusion equations for coupled spin-charge excitations. These equations, which are treated both for weak and strong spin-orbit coupling, allow a due consideration of both the field-induced spin accumulation and spin-coherent excitations. In the weak-coupling limit, the basic results are universal, i.e., they qualitatively agree with findings recently obtained for completely other systems, e.g., the spin transport of small polarons. [29] Whereas the weak-coupling case has received already considerable interest in the literature [19, 20, 30], there are only few considerations of effects due to strong spin-orbit coupling. As shown below completely new and interesting phenomena are predicted to appear. We mention the transition from the diffusive to the ballistic spin transport regime and the occurrence of spin-coherent standing waves. In this paper, emphasis is put on the interesting regime of strong spin-orbit interaction.

II. BASIC THEORY

Coupled spin and charge excitations are treated by an effective-mass Hamiltonian that describes electrons and their spin of a 2DEG with short-range spin-independent elastic scattering on impurities. In addition, the carriers are coupled to each other via spin-orbit interaction of Rashba type. To keep the approach transparent, we postpone the consideration of effects due to external electric and magnetic fields to a forthcoming publication. The second quantized form of the Hamiltonian reads

$$H_0 = \sum_{\boldsymbol{k},\lambda} a^\dagger_{\boldsymbol{k}\lambda} \left[\varepsilon_{\boldsymbol{k}} - \varepsilon_F\right] a_{\boldsymbol{k}\lambda} - \sum_{\boldsymbol{k},\lambda,\lambda'} \left(\hbar\boldsymbol{\omega}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}_{\lambda\lambda'}\right) a^\dagger_{\boldsymbol{k}\lambda} a_{\boldsymbol{k}\lambda'} + u \sum_{\boldsymbol{k},\boldsymbol{k}'} \sum_{\lambda} a^\dagger_{\boldsymbol{k}\lambda} a_{\boldsymbol{k}'\lambda}, \tag{1}$$

where $a^\dagger_{\boldsymbol{k}\lambda}$ ($a_{\boldsymbol{k}\lambda}$) denote the creation (annihilation) operators with in-plane quasi-momentum $\boldsymbol{k}=(k_x,k_y,0)$ and spin λ. In Eq. (1), ε_F denotes the Fermi energy, $\boldsymbol{\sigma}$ the vector of Pauli matrices, and u the strength of the 'white-noise' elastic impurity scattering, which gives rise to the momentum relaxation time τ calculated from

$$\frac{1}{\tau}=\frac{2\pi u^2}{\hbar}\sum_{\boldsymbol{k}'}\delta(\varepsilon_{\boldsymbol{k}'}-\varepsilon_{\boldsymbol{k}}). \tag{2}$$

Other quantities in Eq. (1) are defined by

$$\varepsilon_{\boldsymbol{k}}=\frac{\hbar^2\boldsymbol{k}^2}{2m},\quad \boldsymbol{\omega}_{\boldsymbol{k}}=\frac{\hbar}{m}(\boldsymbol{K}\times\boldsymbol{k}),\quad \boldsymbol{K}=\frac{m\alpha}{\hbar^2}\boldsymbol{e}_z, \tag{3}$$

with α denoting the spin-orbit coupling constant and m the effective mass.

The complete information about all physical quantities of interest are included in the charge and spin components of the spin-density matrix

$$f^\lambda_{\lambda'}(\boldsymbol{k},\boldsymbol{k}'\mid t)=\langle a^\dagger_{\boldsymbol{k}\lambda}a_{\boldsymbol{k}'\lambda'}\rangle_t, \tag{4}$$

which is more conveniently expressed in the following manner

$$f(\boldsymbol{k},\boldsymbol{\kappa}\mid t)=\sum_\lambda f^\lambda_\lambda(\boldsymbol{k},\boldsymbol{\kappa}\mid t),\quad \boldsymbol{f}(\boldsymbol{k},\boldsymbol{\kappa}\mid t)=\sum_{\lambda,\lambda'}f^\lambda_{\lambda'}(\boldsymbol{k},\boldsymbol{\kappa}\mid t)\boldsymbol{\sigma}_{\lambda,\lambda'}. \tag{5}$$

The $\boldsymbol{\kappa}$ dependence, introduced by the replacements $\boldsymbol{k}\to\boldsymbol{k}+\boldsymbol{\kappa}/2$, $\boldsymbol{k}'\to\boldsymbol{k}-\boldsymbol{\kappa}/2$, refers to a possible spatial charge and/or spin inhomogeneity and leads to a coupling between charge- and spin-degrees of freedom. Treating elastic impurity scattering in the Born approximation and keeping only the lowest-order contributions of the spin-orbit interaction in the collision integral, we obtain the following Laplace-transformed kinetic equations [14]

$$sf-\frac{i\hbar}{m}(\boldsymbol{\kappa}\cdot\boldsymbol{k})f-\frac{i\hbar}{m}\boldsymbol{K}\cdot(\boldsymbol{f}\times\boldsymbol{\kappa})=\frac{1}{\tau}(\overline{f}-f)+f_0, \tag{6}$$

$$\begin{aligned}&s\boldsymbol{f}+2(\boldsymbol{\omega}_{\boldsymbol{k}}\times\boldsymbol{f})-\frac{i\hbar}{m}(\boldsymbol{\kappa}\cdot\boldsymbol{k})\boldsymbol{f}+\frac{i\hbar}{m}(\boldsymbol{K}\times\boldsymbol{\kappa})f\\&=\frac{1}{\tau}(\overline{\boldsymbol{f}}-\boldsymbol{f})+\frac{1}{\tau}\frac{\partial}{\partial\varepsilon_{\boldsymbol{k}}}\overline{f\hbar\boldsymbol{\omega}_{\boldsymbol{k}}}-\frac{\hbar\boldsymbol{\omega}_{\boldsymbol{k}}}{\tau}\frac{\partial}{\partial\varepsilon_{\boldsymbol{k}}}\overline{f}+\boldsymbol{f}_0,\end{aligned} \tag{7}$$

where f_0 and $\boldsymbol{f}_0$ denote the initial charge and spin distribution, respectively. The cross-line over $\boldsymbol{k}$-dependent quantities denotes an integration over the angle φ of the in-plane vector $\boldsymbol{k}$ and s is the variable of the Laplace transformation that replaces the time t. The kinetic

Eqs. (6) and (7) have been obtained under the condition $\hbar^2K^2/(m\varepsilon_F) \ll 1$, when the spin contribution on the left-hand side of Eq. (6) can be considered as a small correction.

To proceed we derive coupled spin-charge drift-diffusion equations, which are completely in accordance with the kinetic Eqs. (6) and (7) in the long-wavelength limit. The Eqs. (6) and (7) are written in an more convenient and explicit form as a set of four coupled linear equations

$$\begin{aligned} \sigma f + i\Omega(q_x f_y - q_y f_x) &= R \\ \sigma f_x + 2\Omega\cos(\varphi) f_z - i\Omega q_y f &= R_x + \Omega\sin(\varphi)\frac{\hbar}{\tau}\frac{\partial \overline{f}}{\partial \varepsilon_{\boldsymbol{k}}} \\ \sigma f_y + 2\Omega\sin(\varphi) f_z + i\Omega q_x f &= R_y - \Omega\cos(\varphi)\frac{\hbar}{\tau}\frac{\partial \overline{f}}{\partial \varepsilon_{\boldsymbol{k}}} \\ \sigma f_z - 2\Omega\left[\cos(\varphi) f_x + \sin(\varphi) f_y\right] &= R_z. \end{aligned} \tag{8}$$

Here, we introduced the abbreviations $\Omega = \omega_{\boldsymbol{k}}\tau$, $q_{x,y} = \kappa_{x,y}/k$ and

$$\sigma = \sigma_0 - i\Omega\frac{k}{K}\left[q_x\cos(\varphi) + q_y\sin(\varphi)\right], \quad \sigma_0 = s\tau + 1. \tag{9}$$

The quantities on the right-hand side of Eq. (8) are given by

$$R = \overline{f} + \tau f_0, \quad R_z = \overline{f}_z + \tau f_{0z}, \tag{10}$$

$$R_x = \overline{f}_x + \tau f_{0x} - \frac{\hbar}{\tau}\frac{\partial}{\partial \varepsilon_{\boldsymbol{k}}}\Omega\overline{f\sin(\varphi)}, \quad R_y = \overline{f}_y + \tau f_{0y} + \frac{\hbar}{\tau}\frac{\partial}{\partial \varepsilon_{\boldsymbol{k}}}\Omega\overline{f\cos(\varphi)}. \tag{11}$$

After solving the set of linear Eqs. (8) for the components of the density matrix, the solution is expanded up to second order in q_x and q_y. The final analytical integration over the angle φ results in closed equations for the physical quantities of interest namely $\overline{f}$ and $\overline{\boldsymbol{f}}$. The charge density $\overline{f}$, is expressed by $\overline{f}(\varepsilon_{\boldsymbol{k}}, q \mid s) = n(\varepsilon_{\boldsymbol{k}})F(q \mid s)$, where $n(\varepsilon_{\boldsymbol{k}})$ denotes the Fermi distribution function for carriers in thermal equilibrium. This ansatz is fundamental and completely in line with our basic assumption concerning the relevant time scale of drift-diffusion effects, which should last long enough to guarantee that carriers quickly reestablish the thermal distribution. For these relaxation processes, the energy relaxation time τ_ε is responsible. Usually, the inelastic scattering time is much larger than τ so that the time scale $t > \tau_\varepsilon$ that can be accounted for by the widespread drift-diffusion approach is restricted to $t \gg \tau$ (or $s\tau \ll 1$). The calculation outlined above is straightforward and leads to the result

$$\frac{\partial}{\partial \varepsilon_{\boldsymbol{k}}}\Omega\,\overline{f\sin(\varphi)} = \frac{i\kappa_y}{\sigma_0}\frac{\tau^2 K}{m}(\varepsilon_{\boldsymbol{k}}\overline{f})', \qquad \frac{\partial}{\partial \varepsilon_{\boldsymbol{k}}}\Omega\,\overline{f\cos(\varphi)} = \frac{i\kappa_x}{\sigma_0}\frac{\tau^2 K}{m}(\varepsilon_{\boldsymbol{k}}\overline{f})', \tag{12}$$

where the prime denotes the derivative with respect to $\varepsilon_{\boldsymbol{k}}$. This result is used to derive equations for all components of the density matrix, which decouple into two sets of equations for the components $\overline{f}$ and $i(\boldsymbol{\kappa}\times\overline{\boldsymbol{f}})_z=\overline{f_r}$ on the one hand and $\overline{f}_z$ and $i\boldsymbol{\kappa}\cdot\overline{\boldsymbol{f}}=\overline{f_d}$ on the other hand. We obtain

$$\left[s+D_0(s)\kappa^2\right]\overline{f}+V(s)\overline{f}_r=f_0, \tag{13}$$

$$\left[s+\frac{1}{\tau_{s\perp}(s)}+D_r(s)\kappa^2\right]\overline{f}_r-V_r(s)\kappa^2\overline{f}=f_{r0}, \tag{14}$$

$$\left[s+\frac{1}{\tau_{sz}(s)}+D_z(s)\kappa^2\right]\overline{f}_z-V_z(s)\overline{f}_d=f_{z0}, \tag{15}$$

$$\left[s+\frac{1}{\tau_{s\perp}(s)}+D_d(s)\kappa^2\right]\overline{f}_d-V_d(s)\kappa^2\overline{f}_z=f_{d0}, \tag{16}$$

with the following diffusion coefficients

$$D_0(s)=\frac{D}{\sigma_0^2},\qquad D=v^2\tau/2, \tag{17}$$

$$D_r(s)=\frac{\sigma_0^6+6\sigma_0^4\Omega^2+36\sigma_0^2\Omega^4+48\Omega^6}{\sigma_0^2(\sigma_0^2+4\Omega^2)^2(\sigma_0^2+2\Omega^2)}D, \tag{18}$$

$$D_z(s)=\frac{\sigma_0^2-12\Omega^2}{(\sigma_0^2+4\Omega^2)^2}D,\qquad D_d(s)=\frac{\sigma_0^6-6\sigma_0^4\Omega^2+12\sigma_0^2\Omega^4+16\Omega^6}{\sigma_0^2(\sigma_0^2+4\Omega^2)^2(\sigma_0^2+2\Omega^2)}D, \tag{19}$$

spin relaxation times

$$\tau_{s\perp}(s)=\tau_s\frac{\sigma_0^2+2\Omega^2}{\sigma_0},\qquad \tau_{sz}(s)=\tau_s\sigma_0/2,\qquad 1/\tau_s=4DK^2=2\Omega^2/\tau \tag{20}$$

and coupling terms

$$V(s)=\frac{v}{\sigma_0}\frac{K}{k}\frac{\sigma_0^2+2\Omega^2}{\sigma_0^2+4\Omega^2},\qquad V_r(s)=\frac{v}{\sigma_0}\frac{K}{k}\left[1+\frac{\sigma_0^2+4\Omega^2}{\sigma_0^2+2\Omega^2}\frac{\varepsilon_{\boldsymbol{k}}n'}{n}\right], \tag{21}$$

$$V_z(s)=v\frac{2\Omega}{\sigma_0^2+4\Omega^2},\qquad V_d(s)=v\frac{2\Omega\sigma_0^2}{(\sigma_0^2+4\Omega^2)(\sigma_0^2+2\Omega^2)}. \tag{22}$$

The velocity v is given by $\hbar k/m$. All spin relaxation times are proportional to $1/\tau$ ($1/\tau_s=2\omega_k^2\tau$), which is in accordance with the Dyakonov-Perel spin-relaxation mechanism. The basic solution in Eqs. (13) to (16) allows in principle the consideration of non-Markovian spin-related phenomena because all transport coefficients depend on the variable s of the Laplace transformation. Moreover, in the strong-coupling limit ($\Omega\gg 1$), when the inequality $\tau\gg\tau_s$ is satisfied, the spin diffusion exhibits principal a non-Markovian character. Unfortunately, it turns out that the full consideration of the complicated time dependence in the drift-diffusion Eqs. (13) to (16) is not an passable route. A discussion of this mathematical

subtlety is postponed to a forthcoming paper. In the weak-coupling limit $\Omega \ll 1$, under the condition $\tau \ll \tau_s$, the spin evolution proceeds completely in a Markovian manner.

A striking feature of our basic result is the decomposition of the transport Eqs. (13) to (16) into two sets of coupled equations for (i) the charge density $\overline{f}$ and the transverse spin component $\overline{f}_r$ and (ii) for $\overline{f}_z$ and the longitudinal spin component $\overline{f}_d$. This feature has already been identified for spin-orbit coupled extended electronic states [20] and for hopping transitions of small polarons. [29] As a consequence of this decomposition, the internal effective magnetic field $\boldsymbol{H}$, which is associated with the spin-orbit interaction, is only due to the spin components $\overline{f}_z$ and $\overline{f}_d$ but not to $\overline{f}_r$. From the Maxwell equation, we obtain $\boldsymbol{H}(\boldsymbol{r}) = -\nabla\varphi(\boldsymbol{r})$ with

$$\varphi(\boldsymbol{r}) = \frac{\mu_B}{2\pi}\sum_{\boldsymbol{k}}\int d\boldsymbol{\kappa}\exp\left[-i\kappa_x x - i\kappa_y y - \kappa \mid z \mid\right]\left[\frac{\overline{f}_d(\boldsymbol{k},\boldsymbol{\kappa})}{\kappa} - \frac{z}{\mid z \mid}\overline{f}_z(\boldsymbol{k},\kappa)\right], \tag{23}$$

where μ_B denotes the Bohr magnaton. The measurable magnetic properties are therefore completely determined by Eqs. (15) and (16). With other words, the carrier diffusion leaves the spin-induced magnetization unchanged. The situation changes drastically, when an electric field is applied. In this case, all components of the density matrix couple to each other [29] so that an inhomogeneity in the distribution of charge carriers may also induce a magnetization. Recently this magneto-electric effect has been thoroughly investigated in the literature for semiconductors with spin-orbit interaction.

III. WEAK SPIN-ORBIT COUPLING

The character of spin transport and spin diffusion strongly depends on the strength of the spin-orbit coupling, which is expressed by the dimensionless parameter $\Omega = Kl$. In this Section, we focus on the weak-coupling case $\Omega \ll 1$, when the mean free path l is much shorter than the characteristic length K^{-1} of the spin-orbit coupling. Accordingly, the drift-diffusion Eqs. (13) to (16) are expressed by

$$(s + D\kappa^2)\overline{f} + \frac{\hbar K}{m}f_r = f_0, \tag{24}$$

$$\left(s + \frac{1}{\tau_s} + D\kappa^2\right)\overline{f}_r - \frac{\hbar K}{m}\frac{(\varepsilon_{\boldsymbol{k}}n)'}{n}\kappa^2\overline{f} = f_{r0}, \tag{25}$$

$$\left(s + \frac{2}{\tau_s} + D\kappa^2\right)\overline{f}_z - 4DK\overline{f}_d = f_{z0}, \tag{26}$$

$$\left(s + \frac{1}{\tau_s} + D\kappa^2\right)\overline{f}_d - 4DK\kappa^2\overline{f}_z = f_{d0}, \tag{27}$$

with the spin relaxation time

$$\tau_s = \frac{\tau}{2\Omega^2} = \frac{1}{4DK^2}. \tag{28}$$

These equations formally agree with results obtained for the hopping transport of small polarons. [29] The similarity becomes complete, when the Hall mobility u_H in the latter approach is replaced by the quantity $(e\tau_s/m)(\varepsilon_{\boldsymbol{k}}n)'/n$. Therefore, we conclude that independent of the transport mechanism, the kinetic Eqs. (24) to (27) are universal for weakly spin-orbit coupled carriers. This conclusion is confirmed by similar studies in the literature.[19, 30]

The weak-coupling limit is most important for the hopping regime, where the hopping length must be identified with the mean free path. For the transport of small polarons, this length is the lattice constant a so that the weak-coupling condition $Ka \ll 1$ is almost always satisfied.

Another conclusion follows directly from Eq. (28). For weak spin-orbit coupling, the spin relaxation time τ_s is much larger than τ. Consequently, the charge degree of freedom approaches more quickly the state of equilibrium than the spin degree of freedom so that the frequency response of spin-related quantities to external perturbations does not exhibit a resonance character. According to Eq. (25), an evolution of the transverse spin component $\overline{f}_r$ is expected that follows adiabatically the time variation of $\overline{f}$ when $\tau_s < \tau_\varepsilon$.

The solution of the set of Eqs. (24) to (27) has already been thoroughly studied previously. [19] From the coupled Eqs. (24) and (25), an interesting spin-related effect has been predicted in the literature [20] namely the spontaneous splitting of a density packet into two counter propagating packets with opposite spins. Other results, which are obtained from the universal Eqs. (24) to (27) were derived and discussed in Ref. [29].

Finally, let us add a remark concerning the component of the spin-density vector $\overline{\boldsymbol{f}}$, which is proportional to $\boldsymbol{\kappa}$. To lowest and first-order in $\boldsymbol{\kappa}$, we obtain for the charge density $\overline{f} = n(\varepsilon_{\boldsymbol{k}})/s$. Using this result and accounting for Eqs. (12), (14), and (16), we arrive at a solution

$$\overline{\boldsymbol{f}}_{\boldsymbol{\kappa}} = \frac{-i\boldsymbol{\omega}_{\boldsymbol{\kappa}}\tau}{\sigma_0 s}\left[(\varepsilon_{\boldsymbol{k}}n)' - \frac{2\Omega^2\sigma_0 n}{\sigma_0^2 s\tau + 2\Omega^2(2s\tau+1)}\right] \tag{29}$$

that was derived already in a previous paper. [14] This particular contribution gives rise to a dissipationless spin current in the ground state [14] that reflects the time evolution of the

spin accumulation. Due to the pole structure of this expression, resonances are predicted to occur at characteristic frequencies. [14, 30]

IV. STRONG SPIN-ORBIT COUPLING

For strong spin-orbit coupling $\Omega \gg 1$, the character of the set of Eqs. (13) and (14) for $\overline{f}$ and $\overline{f}_r$ does not change remarkably. We obtain $D_r = 3D/2$ and $\tau_{s\perp} = \tau$, and therefore $\tau_{s\perp} \ll \tau_\varepsilon$ so that in the regime $t \gg \tau$, which is accessible by the drift-diffusion approach, the transverse spin component $\overline{f}_r$ adiabatically follows the evolution of the charge density $\overline{f}$. To confirm this assertion, we analyze Eq. (14) in the strong-coupling limit. Neglecting the small contributions s and $3D\kappa^2/2$ on the left-hand side of Eq. (14), we immediately obtain

$$\overline{f}_r(\varepsilon_{\boldsymbol{k}}, \boldsymbol{r}, t)) = \frac{\hbar K \tau}{mD} \left(1 + \frac{\varepsilon_{\boldsymbol{k}} n'}{n}\right) \frac{\partial}{\partial t} \overline{f}(\varepsilon_{\boldsymbol{k}}, \boldsymbol{r}, t), \tag{30}$$

where $\overline{f}(\varepsilon_{\boldsymbol{k}}, \boldsymbol{r}, t)$ denotes the exact solution of Eq. (13) with $V(s) = 0$.

While the set of Eqs. (13) and (14) for the quantities $\overline{f}$ and $\overline{f}_r$ provide expected solutions also in the strong coupling limit, we observe a dramatic and abrupt change in the behavior of the spin components $\overline{f}_z$ and $\overline{f}_d$ described by Eqs. (15) and (16). As seen from Eq. (19), the diffusion coefficient D_z vanishes at $\Omega = 1/\sqrt{12}$ so that for this coupling strength the physical picture of diffusion completely collapses. With further increasing Ω, the diffusion coefficient D_z becomes negative, which indicates an instability in the spin subsystem implying that the applicability of the kinetic Eqs. (15) and (16) becomes problematic. According to Eq. (28), we have $\tau_s < \tau$ in the strong-coupling limit ($\Omega > 1$). Consequently, the spin diffusion evolves on a time scale that is shorter than the elastic scattering time τ so that the kinetic Eqs. (15) and (16) describe in principle a non-Markovian behavior. Consequently, only in the steady state ($s \to 0$), one can strictly speak about a Markovian spin diffusion in this regime. As shown below, another peculiarity arises namely that the quantities $\overline{f}_z$ and $\overline{f}_d$ rapidly change on a length scale of the order of the mean free path l, a result which does not correspond to the $\boldsymbol{\kappa}$ expansion of Eqs. (6) and (7). Putting all together, we conclude that with increasing coupling strength Ω, a sharp transition appears from a diffusive behavior to a ballistic regime. This transition could be due both to an increasing spin-orbit coupling K and an increasing relaxation time τ.

Since the principal applicability of the drift-diffusion approach for strongly spin-orbit

coupled carriers becomes doubtful, we return to the basic Eqs. (6) and (7) and consider its exact solutions, which are valid in the pure ballistic regime ($\tau \to \infty$) that corresponds to the extreme strong coupling limit. First, we treat the ballistic evolution of a spin packet that is initially ($t = 0$) created at $\boldsymbol{r} = \boldsymbol{0}$. In the limit $\tau \to \infty$, the kinetic Eqs. (6) and (7) are solved by

$$f_z(\boldsymbol{\kappa}, s) = \frac{f_{z0}}{\beta(\boldsymbol{\kappa}, s) + 4\omega_{\boldsymbol{k}}^2/\beta(\boldsymbol{\kappa}, s)}, \qquad \beta(\boldsymbol{\kappa}, s) = s - i(\boldsymbol{\kappa} \cdot \boldsymbol{v}), \tag{31}$$

where $\boldsymbol{v} = \hbar \boldsymbol{k}/m$ denotes the electron velocity and where an additional contribution has been omitted that disappears after the integration over the angle φ. The back transformation to the $\boldsymbol{r}$, t representation is easily accomplished and we get

$$\overline{f}_z(\boldsymbol{r}, t) = f_{z0} \cos(2Kr)\delta(\boldsymbol{r}^2 - \boldsymbol{v}^2 t^2). \tag{32}$$

This solution expresses the pure ballistic character of the spin transport.

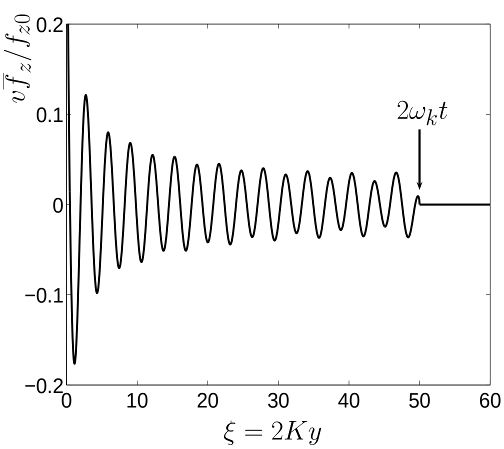

FIG. 1: Spin polarization $v\overline{f}_z/f_{z0}$ as a function of $\xi = 2Ky$ calculated for the half space $y > 0$ from Eq. (33).

The front of the conic spin packet moves with the constant velocity v whereas its amplitude oscillates with a frequency $2Kv$ due to spin rotation. This oscillation manifests itself by a spatial periodic pattern, which in the time domain describes spin precession with the Lamor frequency $2\omega_k$.Let us treat another example namely the half plane $y > 0$ with a given spin polarization at the boundary $y = 0$. This steady-state boundary condition is described by a term f_{z0}/s on the right-hand side of Eq. (7). Again, we start from the solution in Eq. (31), restrict to the y dependence and perform the inverse Fourier and Laplace transformations. The solution

$$\frac{\overline{f}_z}{f_{z0}} = \frac{2\omega_{\boldsymbol{k}} t - \xi}{2\pi v} \Theta(2\omega_{\boldsymbol{k}} t - \xi) \int\limits_0^{1/\xi} d\eta \frac{\cos\left[((2\omega_{\boldsymbol{k}} t - \xi)\eta + 1)\right]}{\sqrt{((2\omega_{\boldsymbol{k}} t - \xi)\eta + 1)^2 - 1}}, \tag{33}$$

manifests a spin front moving with velocity v in the y direction ($\xi = 2Ky$). In contrast to the point source in Eq. (32), one observes non-vanishing oscillatory spin excitations also in the interior. Spin-coherent waves are formed as shown in Fig. 1. The physical origin of this spin

pattern is due to the instability of the system expressed by a negative diffusion coefficient ($D_z < 0$). In this regime, spin diffusion has the tendency to strengthen an initial spin inhomogeneity, whereas the competition with spin relaxation leads to the spatial oscillatory spin pattern.

We point out that the exact solutions of the kinetic Eqs. (6) and (7) in the ballistic regime describe an oscillation spin pattern, which changes on a length scale K^{-1} that is much smaller than the mean free path l ($\Omega = Kl \gg 1$). These exact results derived for the ballistic regime will help to recognize the solution of the drift-diffusion Eqs. (15) and (16) in the limit of strong spin-orbit coupling.

We start our analysis of these equations by calculating its possible eigenmodes. Treating the determinant of Eqs. (15) and (16) in the limit $s \to 0$, we see that there is no oscillatory mode in the weak-coupling regime $\Omega < 1/(3\sqrt{3})$. However, for coupling strengths in the interval $1/(3\sqrt{3}) < \Omega < 1/(2\sqrt{3})$, there are two solutions with vanishing imaginary parts that describe completely undamped oscillations. For even higher spin-orbit interaction $1/(2\sqrt{3}) < \Omega$, only one oscillatory solution is obtained. To illustrate this behavior let us treat a stripe of width $2L$ parallel to the x axis ($-L \leq y \leq L$). A permanent spin polarization should be provided at the boundaries $[\overline{f}_z(y = \pm L, s) = f_{z0}/s]$. For this special case, the inverse Fourier transformation of Eqs. (15) and (16) leads to the following dimensionless differential equations

$$d_z \overline{f}_z'' - \alpha_z \overline{f}_z + \gamma_d F_d = 0, \tag{34}$$

$$d_d F_d'' - \alpha_d F_d - \gamma_z \overline{f}_z'' = 0, \tag{35}$$

with $F_d = \overline{f}_d/K$. The derivative refers to $\xi = 2Ky$ and the parameters are given by

$$\alpha_z = \sigma_0 s\tau + 4\Omega^2, \quad \alpha_d = \sigma_0^2 s\tau + 2\Omega^2(2s\tau + 1), \tag{36}$$

$$\gamma_z = 8\Omega^2 \frac{\sigma_0^2}{\sigma_0^2 + 4\Omega^2}, \quad \gamma_d = \frac{2\sigma_0\Omega^2}{\sigma_0^2 + 4\Omega^2}, \tag{37}$$

$$d_z = 2\sigma_0\Omega^2 \frac{\sigma_0^2 - 12\Omega^2}{(\sigma_0^2 + 4\Omega^2)^2}, \quad d_d = 2\Omega^2 \frac{\sigma_0^6 - 6\sigma_0^4\Omega^2 + 12\sigma_0^2\Omega^4 + 16\Omega^6}{\sigma_0^2(\sigma_0^2 + 4\Omega^2)^2}. \tag{38}$$

With respect to the ξ dependence, the analytic solution of these equations is easily derived. For the boundary conditions $f_{z0} \neq 0$ and $f_{y0} = 0$, we obtain

$$\overline{f}_z = \frac{f_{z0}}{s} \frac{\kappa_1(\alpha_z - d_z\kappa_2^2)\sinh\kappa_2\xi_0\cosh\kappa_1\xi - \kappa_2(\alpha_z - d_z\kappa_1^2)\sinh\kappa_1\xi_0\cosh\kappa_2\xi}{\kappa_1(\alpha_z - d_z\kappa_2^2)\sinh\kappa_2\xi_0\cosh\kappa_1\xi_0 - \kappa_2(\alpha_z - d_z\kappa_1^2)\sinh\kappa_1\xi_0\cosh\kappa_2\xi_0}, \tag{39}$$

with $\xi_0 = 2KL$. $\kappa_{1,2}$ are solutions of the biquadratic equation

$$(d_z\kappa^2 - \alpha_z)(d_d\kappa^2 - \alpha_d) + \gamma_z\gamma_d\kappa^2 = 0, \tag{40}$$

with the plus sign. The remaining inverse Laplace transformation requires a numerical integration.

First, let us treat the steady-state solution ($s = 0$) of Eq. (39).

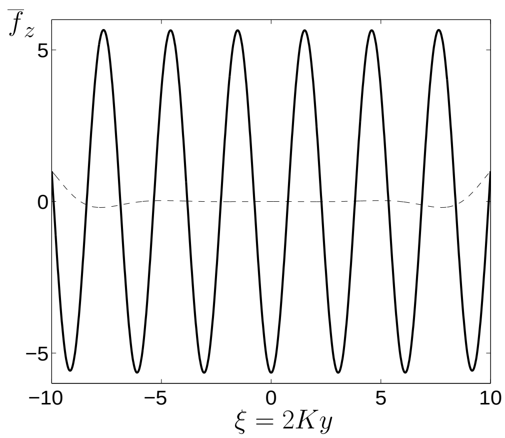

FIG. 2: Spin polarization $v\overline{f}_z$ as a function of $\xi = 2Ky$ for $\Omega = 0.1$ (dashed line) and $\Omega = 1$ (solid line).

Figure 2 shows the spin polarization $\overline{f}_z$ as a function of ξ. As expected, for weak spin-orbit coupling (dashed line), the spin polarization gradually decreases into the interior of the stripe and vanishes completely at the center. A completely other behavior exhibits the solid line calculated for $\Omega = 1$. A spin-coherent standing wave appears for strong spin-orbit coupling. The period of this pattern is comparable with the mean-free path l ($\Omega = Kl = 1$). Such rapid oscillations are in principle beyond the accessibility of the drift-diffusion approach. However, as already mentioned above, similar oscillations appear also in the ballistic regime. Moreover, recently it has been stated that the macroscopic transport equations work well even when the spin-diffusion length is comparable to the mean-free path. [31]

In the strong-coupling limit $\Omega \gg 1$, the solution in Eq. (39) simplifies to

$$\overline{f}_z(k, y, s) = \frac{f_{z0}}{s}\frac{\cos(4\Omega\xi/\sqrt{6})}{\cos(4\Omega\xi_0/\sqrt{6})}. \tag{41}$$

The period of this oscillation [given by $l\sqrt{6}\pi/(4(Kl)^2)$] is much smaller than the mean-free path l. In addition, the spin polarization $\overline{f}_z$ in Eq. (41) diverges, when the half width L of the stripe satisfies the resonance condition $(2n+1)\pi/2 = 4\Omega\xi_0/\sqrt{6}$ with n being any integer. The appearance of these divergencies indicates that elastic scattering alone cannot always balance the tendency of mutual spin alignment in the strong spin-orbit coupling regime. To avoid this defect of the approach, obviously both elastic and inelastic scattering have to be treated on a full microscopic level.

To give a rough idea of the temporal evolution of spin-coherent standing waves, we treat Eq. (39) in the quasi-stationary regime ($s\tau \ll 1$, i.e., $\sigma_0 = 1$) and perform the inverse Laplace transformation by a numerical integration. Figure 3 shows an example for $\overline{f}_z$ as a function of ξ and t/τ. Initially, at $t = 0$, there is only the provided spin polarization f_{z0} at the boundaries. The time evolution starts with an intricate dependence that is by no means correctly reproduced by the drift-diffusion approach because it appears at a time scale $t \lesssim \tau$. After this short period of rapid spatio-temporal changes, the periodic spin pattern gradually emerges. Whereas the gross features of this picture are certainly reliable, a detailed analysis of the time evolution requires an approach based on a microscopic theory.

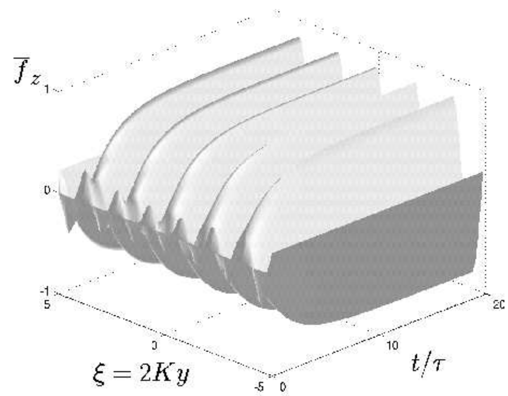

FIG. 3: Spin polarization $\overline{f}_z$ as a function of t/τ and $\xi = 2Ky$ for $\Omega = 2$ and $f_{z0} = 0.2$.

V. SUMMARY

Starting from microscopic quantum-kinetic equations for the spin-density matrix, we derived drift-diffusion equations for a 2DEG with Rashba spin-orbit interaction. The approach is valid in the weak as well as strong coupling regime. In the absence of external electric and magnetic fields, the set of equations decouples. The first set of equations relates the charge density to the transverse in-plane spin component. Another set of equations links the out-of plane spin component with the longitudinal in-plane spin contribution. Whereas both sets of equations have an universal character in the weak scattering limit, they exhibit interesting peculiarities for strong spin-orbit coupling. In the latter case, the spin relaxation time τ_s becomes shorter than the elastic scattering time τ. Consequently, spin effects have a principal non-Markovian character in this regime. Moreover, the diffusion coefficient D_z of the $\overline{f}_z$-$\overline{f}_d$ spin channel changes its sign with increasing spin-orbit coupling $\Omega = Kl$. This dependence gives rise to an instability in the spin subsystem ($D_z = 0$) and to an undamped oscillatory spin pattern ($D_z < 0$). The wavelength of the long-lived coherent spin rotation is smaller than the mean-free path. This fact conflicts with basic assumptions of the drift-diffusion approach, which is only applicable for times much longer than the elastic scattering

time and for diffusion lengths much larger than the mean-free path. The approach signals the existence of an instability similar to the recently studied transition from a uniform to a nonuniform ground state in the Rashba model. [32] The rapid spatial variations of the out-of plane spin polarization gives rise to microscopic circulating currents via the induced magnetic field. Within a self-consistent schema, one expects that the retroaction of these currents on spins may lead to a stabilization of the spin subsystem and to a finite damping of spin oscillations.

In order to appreciate the unusual results obtained from the drift-diffusion equations in the strong-coupling limit, an exact treatment of the original quantum-kinetic equations is desirable. For strong spin-orbit coupling ($\Omega \gg 1$), the spin relaxation time is much smaller than the scattering time. Therefore, most interesting is the consideration of the ballistic spin regime. Exact analytical solutions of the kinetic equations confirm the existence of a long-lived oscillating spin pattern. This confirmation strongly suggests that spin-coherent standing waves in a semiconducting stripe are indeed serious solutions of the drift-diffusion equations. Moreover, a recent alternative approach based on Monte-carlo simulations also arrived at the conclusion that spin-coherent standing waves have an extremely long spin-relaxation time. [28]

We conclude that the prediction of an oscillatory spin pattern in semiconductors with strong spin-orbit coupling of the Rashba type seems to be interesting both from a theoretical and experimental point of view. Further progress is expected from a due treatment of the quantum-kinetic equations for the density matrix beyond the pure ballistic regime ($\tau \to \infty$). In addition, external electric and magnetic fields sensitively influence spin coherent waves by mixing all components of the density matrix. The experimental verification of the predicted long-lived spin-coherent waves could be useful for future spintronic applications.

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Partial financial support by the Deutsche Forschungsgemeinschaft and the Russian Foundation of Basic Research under the grant number 05-02-04004 is gratefully acknowledged.

[1] S. Murakami, N. Nagaosa, and S. C. Zhang, Science **301**, 1348 (2003).

[2] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004).

[3] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004).

[4] J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).

[5] V. Sih, W. H. Lau, R. C. Myers, V. R. Horowitz, A. C. Gossard, and D. D. Awschalom, Phys. Rev. Lett. **97**, 096605 (2006).

[6] E. I. Rashba, Phys. Rev. B **68**, 241315 (2003).

[7] O. V. Dimitrova, Phys. Rev. B **71**, 245327 (2005).

[8] Q. Sun and X. C. Xie, Phys. Rev. B **72**, 245305 (2005).

[9] S. Zhang and Z. Yang, Phys. Rev. Lett. **94**, 066602 (2005).

[10] A. Khaetskii, Phys. Rev. Lett. **96**, 056602 (2006).

[11] E. I. Rashba, Physica E **34**, 31 (2006).

[12] J. Shi, P. Zhang, D. Xiao, and Q. Niu, Phys. Rev. Lett. **96**, 076604 (2006).

[13] N. Sugimoto, S. Onoda, S. Murakami, and N. Nagaosa, Phys. Rev. B **73**, 113305 (2006).

[14] V. V. Bryksin and P. Kleinert, Phys. Rev. B **73**, 165313 (2006).

[15] E. Mishchenko and B. I. Halperin, Phys. Rev. B **68**, 045317 (2003).

[16] Y. Qi and Z. Zhang, Phys. Rev. B **67**, 052407 (2003).

[17] M. Q. Weng and M. W. Wu, Phys. Rev. B **68**, 075312 (2003).

[18] S. Saikin, J. Phys.: Condens. Matter **16**, 5071 (2004).

[19] A. A. Burkov, A. S. Nunez, and A. H. MacDonald, Phys. Rev. B **70**, 155308 (2004).

[20] B. A. Bernevig, X. Yu, and S. C. Zhang, Phys. Rev. Lett. **95**, 076602 (2005).

[21] O. Bleibaum, Phys. Rev. B **72**, 075366 (2005).

[22] O. Bleibaum, Phys. Rev. B **73**, 035322 (2006).

[23] F. X. Bronold, A. Saxena, and D. L. Smith, Phys. Rev. B **70**, 245210 (2004).

[24] M. Hruska, S. Kos, S. A. Crooker, A. Saxena, and D. L. Smith, Phys. Rev. B **73**, 075306 (2006).

[25] J. Wang, K. S. Chan, and D. Y. Xing, Phys. Rev. B **73**, 033316 (2006).

[26] O. Bleibaum, Phys. Rev. B **74**, 113309 (2006).

[27] V. M. Galitski, A. A. Burkov, and S. D. Sarma, Phys. Rev. B **74**, 115331 (2006).

[28] Y. V. Pershin, Phys. Rev. B **71**, 155317 (2005).

[29] V. V. Bryksin, H. Böttger, and P. Kleinert, Phys. Rev. B **74**, 235302 (2006).

[30] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).

[31] D. R. Penn and M. D. Stiles, Phys. Rev. B **72**, 212410 (2005).

[32] X. G. Wu, L. J. Xu, and H. Z. Zheng, Phys. Rev. B **74**, 165309 (2006).